The History of Euler's Disk<sup>®</sup> (Spinning and Rolling Disk) "...Namely, because the shape of the whole universe is most perfect..." L. Euler

Over the past 20+ years, many have asked why the toy is named "Euler's Disk." To answer this rather simple question, let us begin with a short story that explains the history of how the toy was created and what finally convinced me to name the device "Euler's Disk."

The early days and serendipity:

Around 1987, while working at a large aerospace company in southern California (the best portion of my engineering career) I remember spinning coins on the cafeteria table when bored or thinking about some particular problem. Many millions of people around the world spin coins on table tops everyday and let them wander about; I was (and am) no different, and this story should end here. But, at this point, I can not help but steal a phrase from Horace Walpole, I was soon to become an "Engineer of Serendip." One day, I forget the exact date (it was after lunch), I was sitting at my desk waiting for a shipment of metrology tools to arrive (my electronic calipers were constantly missing) for a rather boring micromachining experiment. When the box finally arrived, I tore it open and found it contained 12 shiny, razor sharp polishing chucks – a mistake. By habit, I started spinning these disks on my desk (hard desk = long spolling time). A few of these disks hummed so loudly that a fellow engineer (Fred S) entered the room and said "What is going on Mr. Bendik?" After ~30 seconds of spolling (spin and roll) something struck me - from what I know all it takes is a rather long spoll (~40 seconds) to suddenly change how one feels about spolling a simple disk on a desk - a sense that something very interesting is going on -a sense that forced me to explore spolling to the nth-degree. I became obsessed with spolling disks - constantly trying to create disks that would spoll for longer periods of time.

Early prototype disks included large, rusty (~10 pound) lathe weights that were lying about the machine shop floor and polishing chucks. Larger prototypes rolled for ~30 seconds and made loud and distinct sounds when "spolling" abruptly ended (occasionally causing damage to the concrete floor). I usually waited until lunch or evenings to spin the disks at work (a fellow engineer and friend, Rich Wyles, was my look-out). These large heavy disks were fun to spoll and represent some early experiments but they really were not worth mentioning to any mechanical engineer or anyone else, at least not yet. Over time, my friend and I were faced with solving (or surrendering to) several interesting optimization problems (i.e., finding maxima and minima of scalar quantities). Leonard Euler worked on these variational principles in the eighteen century.

## Variations of a theme:

One of the company's best FPA designers was an avid supporter of my attempt to create the longest "spolling" time. Together we tried spinning the toy on every possible surface we could find. Eventually, we found some glass table tops stored in the old warehouse (it was a great warehouse). We quickly discovered that metal on glass brought over 60 second spins - a record at that time. Over the years, my friends (Rich Wyles and Larry Shaw) and I solved several problems related to the toy's motion – we studied how the toy loses energy as it moves on a given surface, the motion on both flat and curved surfaces – the path traced, we looked into skipping, sliding, slipping, and rolling motion. We set up a matrix of experiments to try, built strange prototypes (w/wo holes, magnetic, aluminum, wood), and recorded results. We plotted results to obtain optimums, even considering some unobvious constraints. In fact, a prototype for the mirror base was found by accident on a trip to La Jolla one sunny afternoon - sitting in a "for sale" bin at a corner drug store. The progression of experiments that led to a desk size prototype that can spoll for > 4 minutes involved considerations for just about every aspect of the physical characteristics found on the commercial version (the current commercial version has a design time of ~3 minutes for several interesting reasons as well).

For several years, I kept the toy to myself and a few friends and never planned to build a version for distribution (the path toward commercializing the toy is a separate story but obviously follows logically from this one). The toy really exhibits a rather complex motion - typically, not appreciated by the untrained eye. And still, after all this work, there is more to be discovered...the motion of this "spolling" disk and variations are studied continuously – please see the website publication section.

Finally, why "Euler's Disk": Eventually, I needed to call the spolling disk something other than "spolling disk." Over time, several years before the toy was commercialized, I realized the motion of the disk was probably given some thought by Euler. Reading the literature on Euler, one finds he was very interested in the math and physics of "spolling" (spin & roll) rigid bodies (hoops for example) - hence the name "Euler's Disk." Anyone setting up the interesting (coupled) physics equations related to the motion of the toy will most likely make use Eulerian angles and Euler's equations of motion to yield some classic results. At one point, we were working on a problem whose solution required solving the Laplace equation - actually first derived by Euler in 1752. In 2000, The Journal of Nature published a short article on Euler's Disk by H. K. Moffatt, a fluid dynamicist at the University of Cambridge. Moffatt used elements of fluid mechanics to calculate how energy "might" be lost near the end of spolling (Moffat's unique calculation and discussion on a finite time singularity has been the subject of several studies). The equation for a perfect fluid, modified by Navier and Stokes with viscous effects, is the work of Leonard Euler (his 310th birthday was in 2017).

©JJBendik2007/16/17/19/20 Epilogue: Euler's Disk has been finished for some time now and I am satisfied with that. I have always thought that Euler's Disk displays a beautiful and elegant motion. Over the years it might just become a "classic." In the end, after a bit of hard work, the toy has brought happiness and intrigue to many people all over the world. So in the end, if you're happy spinning coins on tabletops and never plan to purchase a toy like Euler's Disk - then you are a lot like me...many years ago.

Joseph Bendik

Brief but important credits:

1) "Introduction to Space Dynamics," William Thomson.

2) Chaotic motion for rolling points of contact - Joseph Bendik, Hughes Aircraft Co.

3) Nyquist sampling / strobe light effects - Richard Wyles, Raytheon.

4) Dimensional Analysis - Larry Shaw inventor of Astrojax.

5) Acoustics and the rolling point of contact - Dr. Michael Brooks – NASA.

6) Pseudorotation (see publications) - David Harris, UCSB.

7) Pulsar motion – (see publications) - Lar's Bildsten, UCSB, KITP.

8) Helium experiments - Joseph Bendik - Hughes Aircraft Co. (1989).

9) Vacuum Experiments - David Harris and Keith Moffatt – UCSB (May, 2000).

10) CAD layouts - Mark Stenholm.

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